International Baccalaureate
Baccalauréat International
Bachillerato Internacional

22117406

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Thursday 5 May 2011 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 23]*

Part A

A university required all Science students to study one language for one year. A survey was carried out at the university amongst the 150 Science students. These students all studied one of either French, Spanish or Russian. The results of the survey are shown below.

	French	**Spanish**	**Russian**
Female	9	29	12
Male	31	40	29

Ludmila decides to use the χ^2 test at the 5 % level of significance to determine whether the choice of language is independent of gender.

(a) State Ludmila's null hypothesis. *[1 mark]*

(b) Write down the number of degrees of freedom. *[1 mark]*

(c) Find the expected frequency for the females studying Spanish. *[2 marks]*

(d) Use your graphic display calculator to find the χ^2 test statistic for this data. *[2 marks]*

(e) State whether Ludmila accepts the null hypothesis. Give a reason for your answer. *[2 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B

At the end of the year, only seven of the female Science students sat examinations in Science and French.

The marks for these seven students are shown in the following table.

Science (S)	23	51	56	62	12	73	72
French (F)	65	45	45	40	70	36	30

(a) Draw a labelled scatter diagram for this data. Use a scale of 2 cm to represent 10 marks on the *x*-axis (S) and 10 marks on the *y*-axis (F).   *[4 marks]*

(b) Use your graphic display calculator to find

  (i) $\overline{S}$, the mean of S;

  (ii) $\overline{F}$, the mean of F.   *[2 marks]*

(c) Plot the point $\text{M}\left(\overline{S}, \overline{F}\right)$ on your scatter diagram.   *[1 mark]*

(d) Use your graphic display calculator to find the equation of the regression line of F on S.   *[2 marks]*

(e) Draw the regression line on your scatter diagram.   *[2 marks]*

Carletta's mark on the Science examination was 44. She did not sit the French examination.

(f) Estimate Carletta's mark for the French examination.   *[2 marks]*

Monique's mark on the Science examination was 85. She did not sit the French examination. Her French teacher wants to use the regression line to estimate Monique's mark.

(g) State whether the mark obtained from the regression line for Monique's French examination is reliable. Justify your answer.   *[2 marks]*

2. *[Maximum mark: 14]*

Give all your numerical answers correct to two decimal places.

On 1 January 2005, Daniel invested 30 000 AUD at an annual **simple** interest rate in a *Regular Saver* account. On 1 January 2007, Daniel had 31 650 AUD in the account.

(a) Calculate the rate of interest. *[3 marks]*

On 1 January 2005, Rebecca invested 30 000 AUD in a *Supersaver* account at a nominal annual rate of 2.5 % **compounded annually**.

(b) Calculate the amount in the *Supersaver* account after two years. *[3 marks]*

(c) Find the number of complete years since 1 January 2005 it would take for the amount in Rebecca's account to exceed the amount in Daniel's account. *[3 marks]*

On 1 January 2007, Daniel reinvested 80 % of the money from the *Regular Saver* account in an *Extra Saver* account at a nominal annual rate of 3 % **compounded quarterly**.

(d) (i) Calculate the amount of money reinvested by Daniel on the 1 January 2007.

  (ii) Find the number of complete years it will take for the amount in Daniel's *Extra Saver* account to exceed 30 000 AUD. *[5 marks]*

3. *[Maximum mark: 18]*

Part A

A geometric sequence has 1024 as its first term and 128 as its fourth term.

(a)   Show that the common ratio is $\dfrac{1}{2}$.   *[2 marks]*

(b)   Find the value of the eleventh term.   *[2 marks]*

(c)   Find the sum of the first eight terms.   *[3 marks]*

(d)   Find the number of terms in the sequence for which the **sum** first exceeds 2047.968.   *[3 marks]*

Part B

Consider the arithmetic sequence $1, 4, 7, 10, 13, \ldots$

(a)   Find the value of the eleventh term.   *[2 marks]*

(b)   The sum of the first n terms of this sequence is $\dfrac{n}{2}(3n-1)$.

    (i)   Find the sum of the first 100 terms in this arithmetic sequence.

    (ii)   The sum of the first n terms is 477.

        (a)   Show that $3n^2 - n - 954 = 0$.

        (b)   Using your graphic display calculator or otherwise, find the number of terms, n.   *[6 marks]*

4. *[Maximum mark: 15]*

The diagram represents a small, triangular field, ABC, with $BC = 25\,m$, angle $BAC = 55°$ and angle $ACB = 75°$.

diagram not to scale
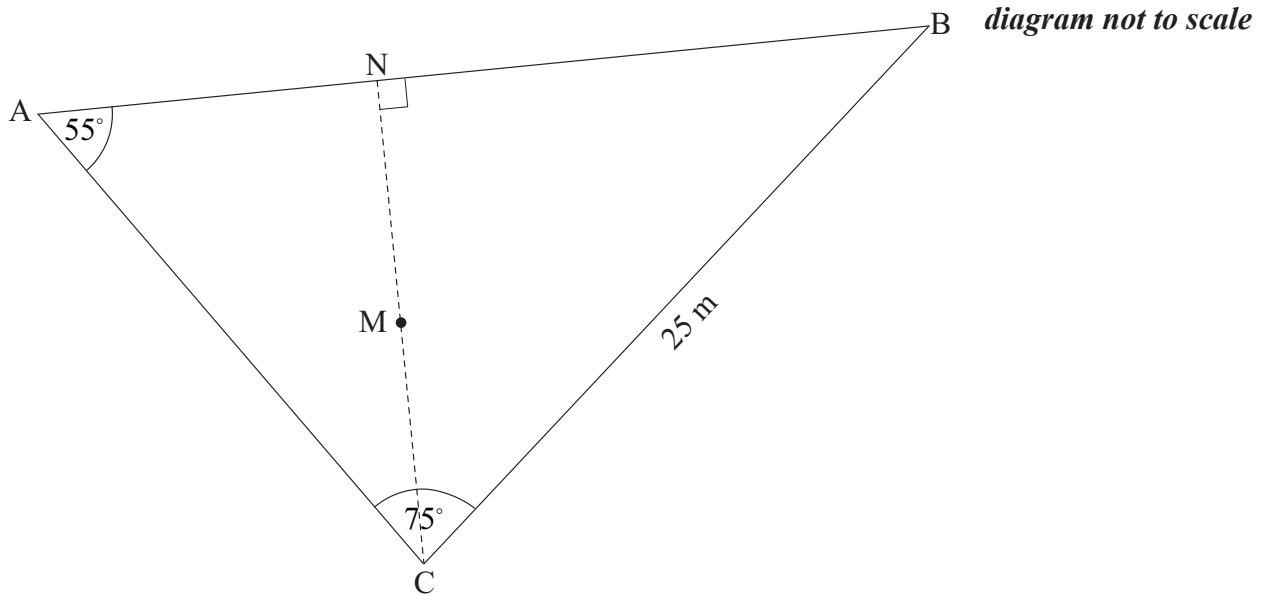
(a) Write down the size of angle ABC. *[1 mark]*

(b) Calculate the length of AC. *[3 marks]*

(c) Calculate the area of the field ABC. *[3 marks]*

N is the point on AB such that CN is perpendicular to AB. M is the midpoint of CN.

(d) Calculate the length of NM. *[3 marks]*

A goat is attached to one end of a rope of length 7 m. The other end of the rope is attached to the point M.

(e) Decide whether the goat can reach point P, the midpoint of CB. Justify your answer. *[5 marks]*

5.   *[Maximum mark: 20]*

The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \dfrac{6}{x}$, $x \neq 0$.

(a)   Write down the equation of the vertical asymptote.   *[2 marks]*

(b)   Find $f'(x)$.   *[3 marks]*

(c)   Find the gradient of the graph of the function at $x = -1$.   *[2 marks]*

(d)   Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer.   *[2 marks]*

(e)   Sketch the graph of $f(x)$ for $-10 \leq x \leq 10$ and $-20 \leq y \leq 20$.   *[4 marks]*

P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$.

(f)   Using your graphic display calculator, write down the coordinates of

(i) P_1;

(ii) P_2.   *[4 marks]*

(g)   Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leq x \leq 10$.   *[3 marks]*